Chapter 13: Oscillatory Motion
From Part Two of the Textbook
Tuesday March 31st

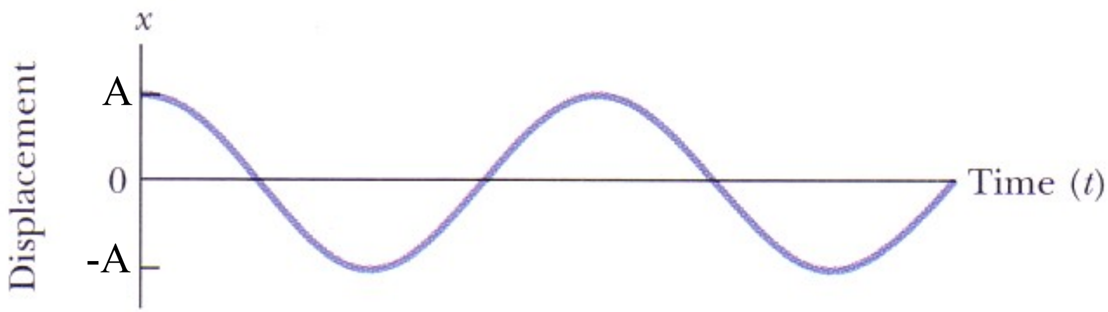
- Review: Simple Harmonic Motion (SHM)
- Review: Hooke's Law and SHM
- Energy in SHM
- The Simple Pendulum
- Damped Harmonic Motion
- Examples, demonstrations and *i*clicker
- If time at end (unlikely), one more solution to mid-term

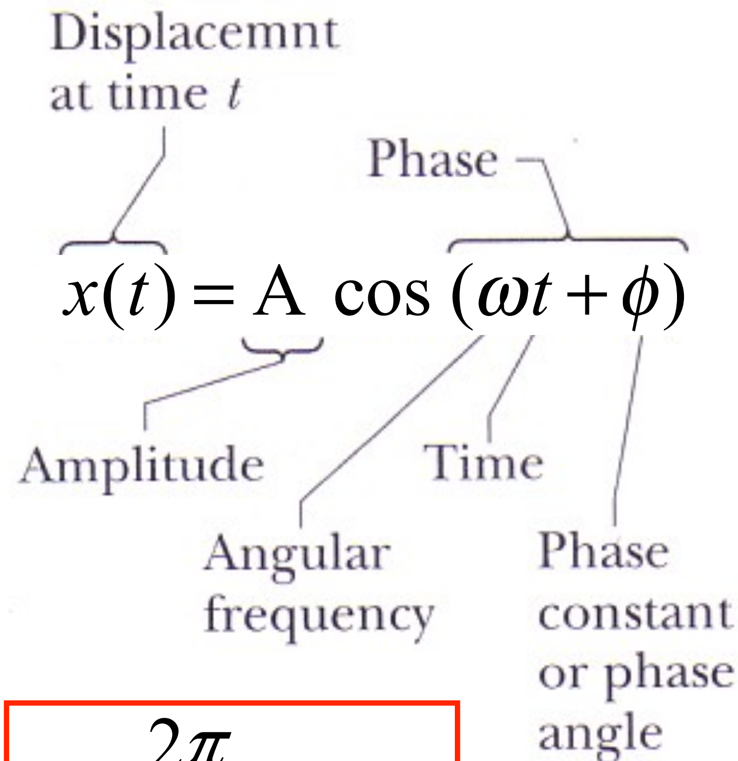
- Final Mini Exam next week on Thursday (April 9)
- Will cover oscillations and waves (this week/next LONCAPA)

Reading: up to page 218 in Ch. 13

Review: Simple Harmonic Motion

- The simplest possible form of harmonic motion is called Simple Harmonic Motion (SHM).
- This term implies that the periodic motion is a sinusoidal (or cosine) function of time,





$$x(t) = A\cos(\omega t + \phi)$$

- We can find the relationship between ω and T in the following way.
- Since the motion repeats itself,

$$\omega(t+T) = \omega t + 2\pi \quad \Rightarrow \omega T = 2\pi$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

- A and ϕ determined by the initial conditions of the oscillation.
- The frequency ω is independent of A and ϕ.

The velocity and acceleration of SHM

Velocity:

$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}\left[A\cos\left(\omega t + \phi\right)\right]$$

$$v_{max} = \omega A$$

$$v(t) = -\omega A\sin\left(\omega t + \phi\right) = -v_{max}\sin\left(\omega t + \phi\right)$$

- The positive quantity ωA is the maximum velocity v_{max} (amplitude of v)

Acceleration:

$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}\left[-\omega A\sin\left(\omega t + \phi\right)\right]$$

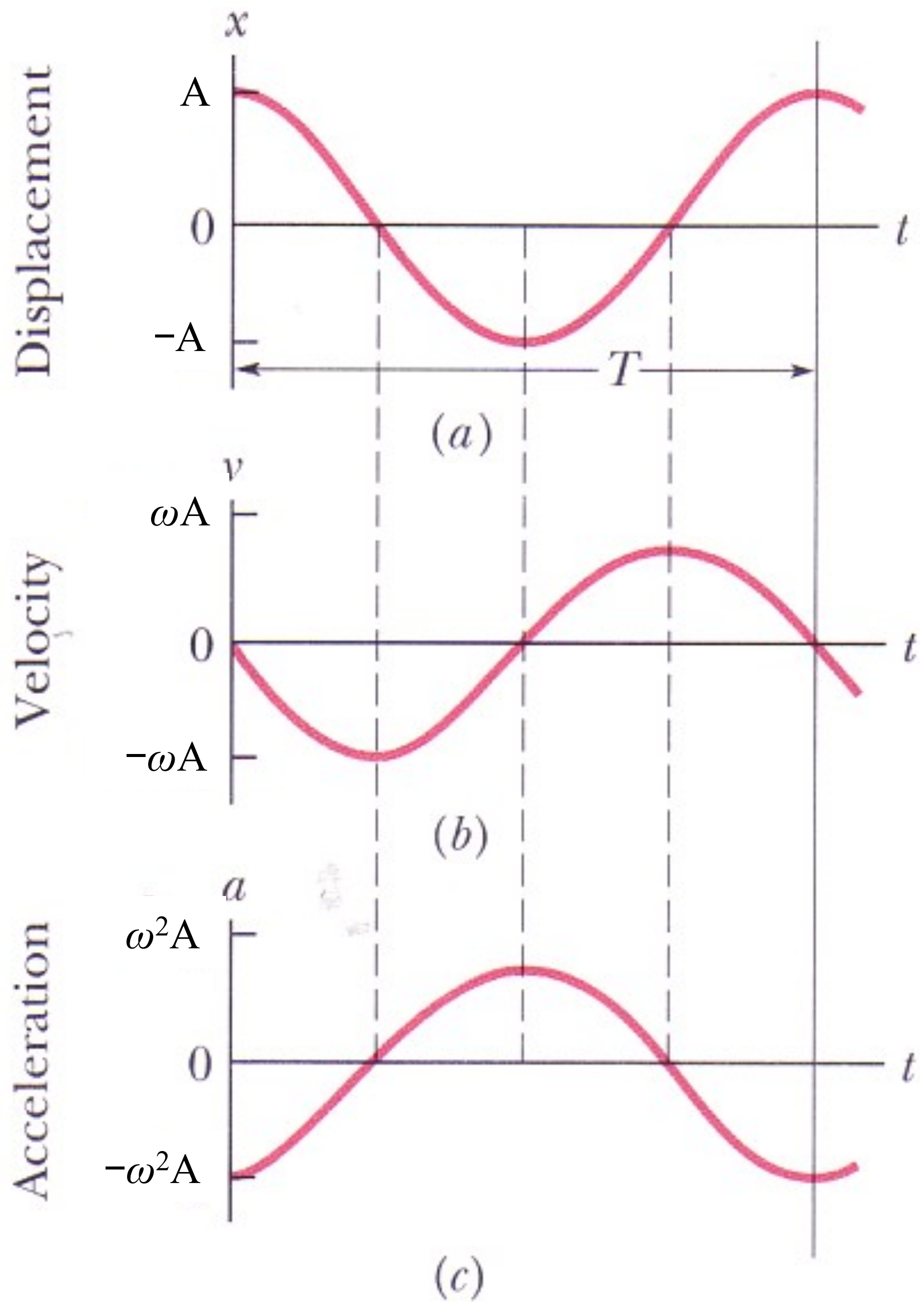
$$a_{max} = \omega^2 A$$

$$a(t) = -\omega^2 A\cos\left(\omega t + \phi\right) = -a_{max}\cos\left(\omega t + \phi\right)$$

$$a(t) = -\omega^2 x(t)$$

In SHM, the acceleration is proportional to the displacement but opposite in sign; the two quantities are related by the square of the angular frequency

The velocity and acceleration of SHM



- Notice the $\pi/2$ phase shift between the velocity and the displacement.

- The acceleration is opposite in sign to the displacement.
- One can also relate them by a π phase shift.

The force law for SHM

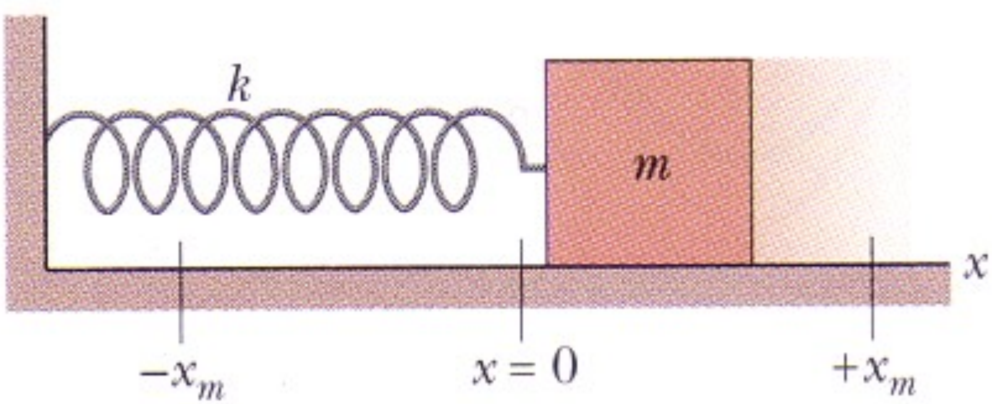
$$F = ma = m(-\omega^2 x) = -(m\omega^2)x$$

•Note: SHM occurs in situations where the force is proportional to the displacement, and the proportionality constant $(-m\omega^2)$ is negative, *i.e.*,

$$F = -kx$$

•This is very familiar - it is Hooke's law.

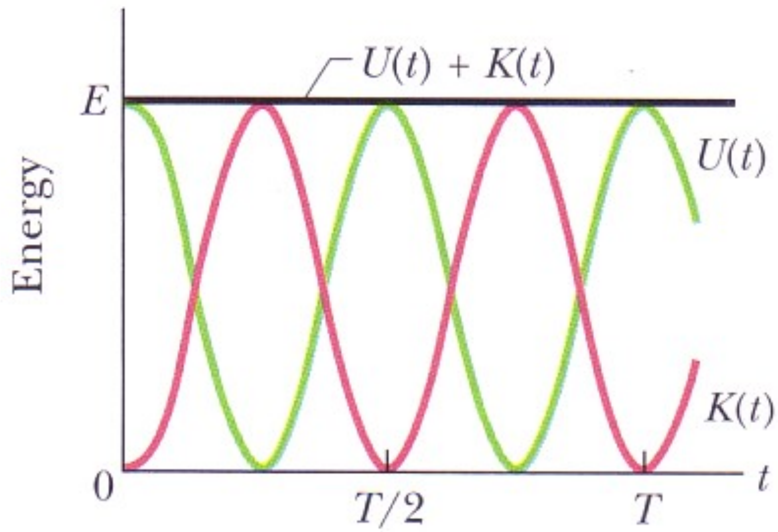
SHM is the motion executed by a particle of mass *m* subjected to a force that is proportional to the displacement of the particle but of opposite sign.



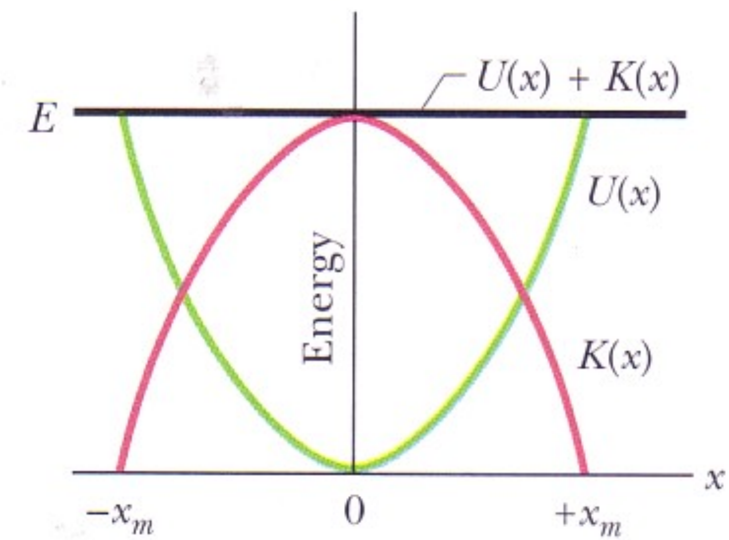
$$k = m\omega^2$$

$$\omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Energy in SHM



(a)



(b)

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2\cos^2(\omega t + \phi)$$

$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2A^2\sin^2(\omega t + \phi)$$

But $$\omega^2 = \frac{k}{m}$$

Thus, $$K(t) = \tfrac{1}{2}kA^2\sin^2(\omega t + \phi)$$

$$E = U + K$$

$$= \tfrac{1}{2}kA^2\left[\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)\right]$$

$$\cos^2\alpha + \sin^2\alpha = 1$$

So: $$E = U + K = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\max}^2$$

Simple Pendulum

Torque about P:

$$\tau = -mgr_{\perp} = -mgL\sin\theta$$

$$= I\alpha = I\frac{d^2\theta}{dt^2}$$

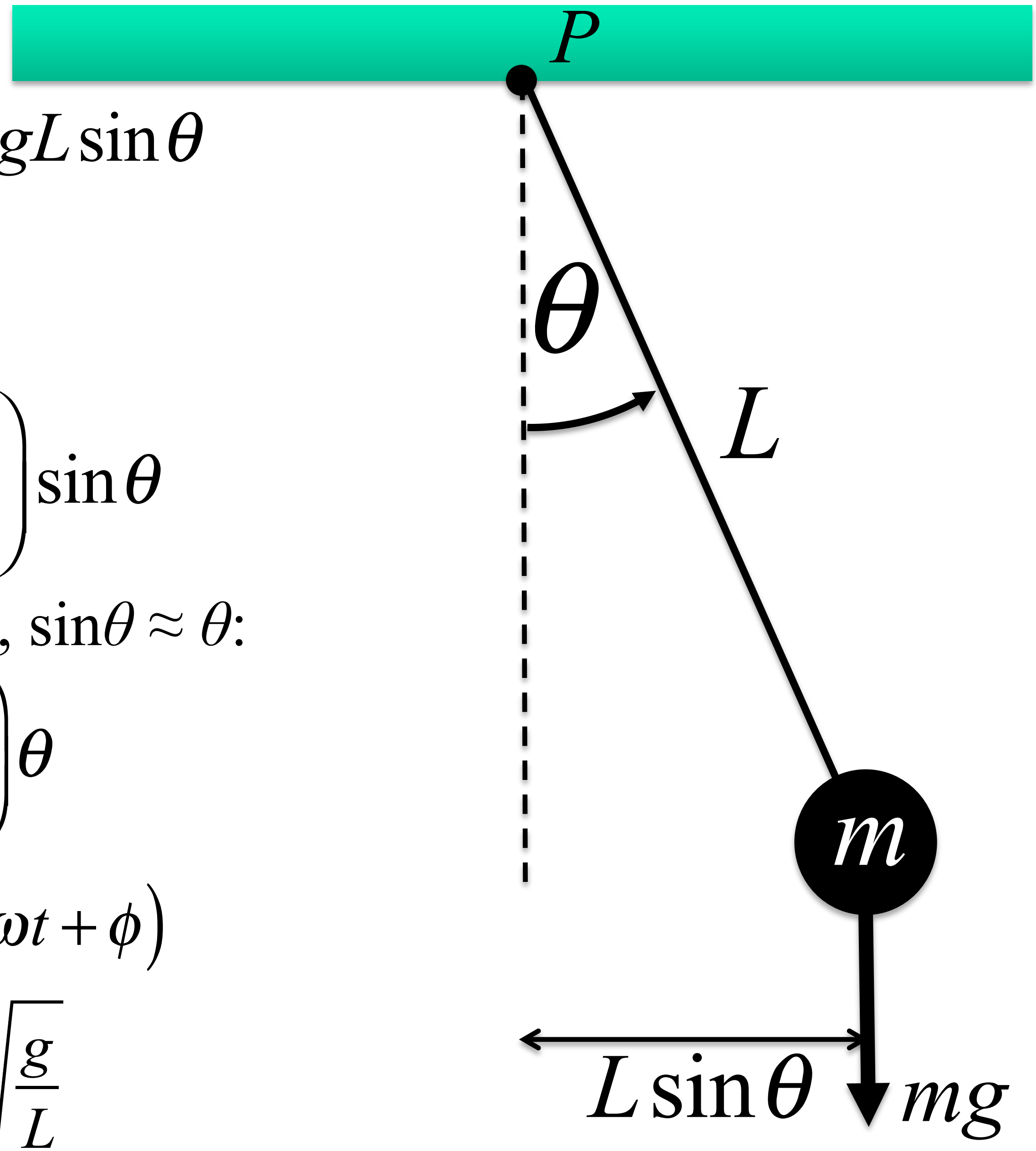
$$\Rightarrow \frac{d^2\theta}{dt^2} = -\left(\frac{mgL}{I}\right)\sin\theta$$

For small displacements, $\sin\theta \approx \theta$:

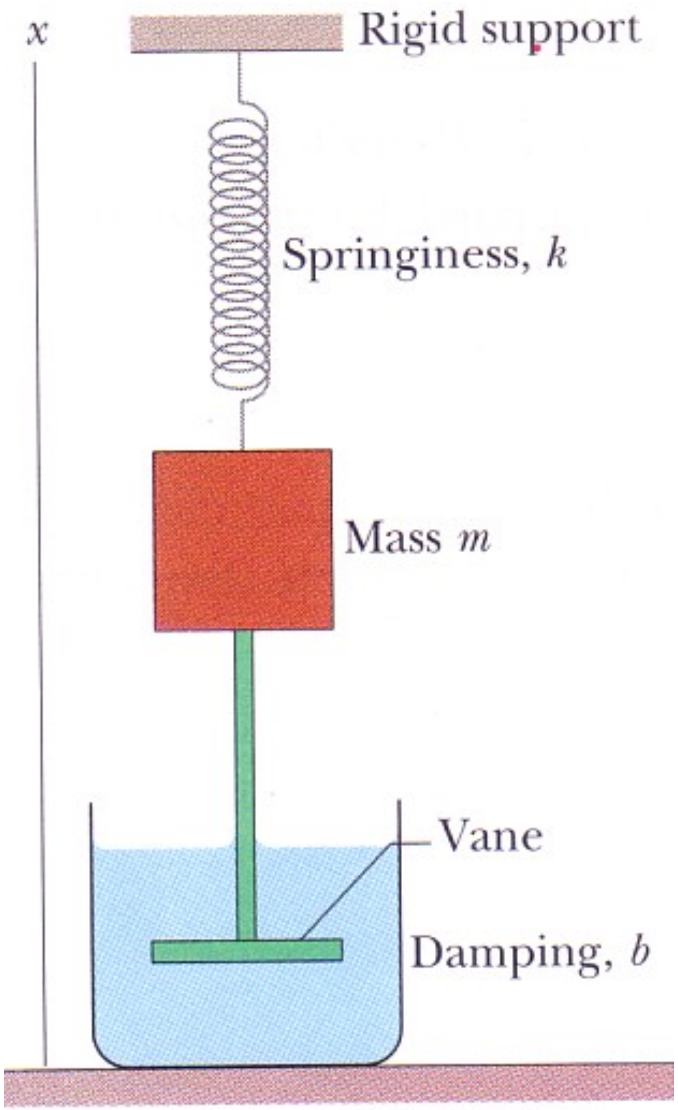
$$\frac{d^2\theta}{dt^2} \approx -\left(\frac{mgL}{I}\right)\theta$$

$$\Rightarrow \quad \theta(t) = \theta_{\max}\cos(\omega t + \phi)$$

$$\& \quad \omega = \sqrt{\frac{mgL}{I}} = \sqrt{\frac{g}{L}}$$



Damped Simple Harmonic Motion



- When the motion of an oscillator is reduced by an external force, the oscillator and its motion are said to be damped.
- Let us assume that the liquid in the figure (left) exerts a constant damping force that is proportional in magnitude to the velocity (like air resistance), and opposite in sign, *i.e.*,

$$F_d = -bv$$

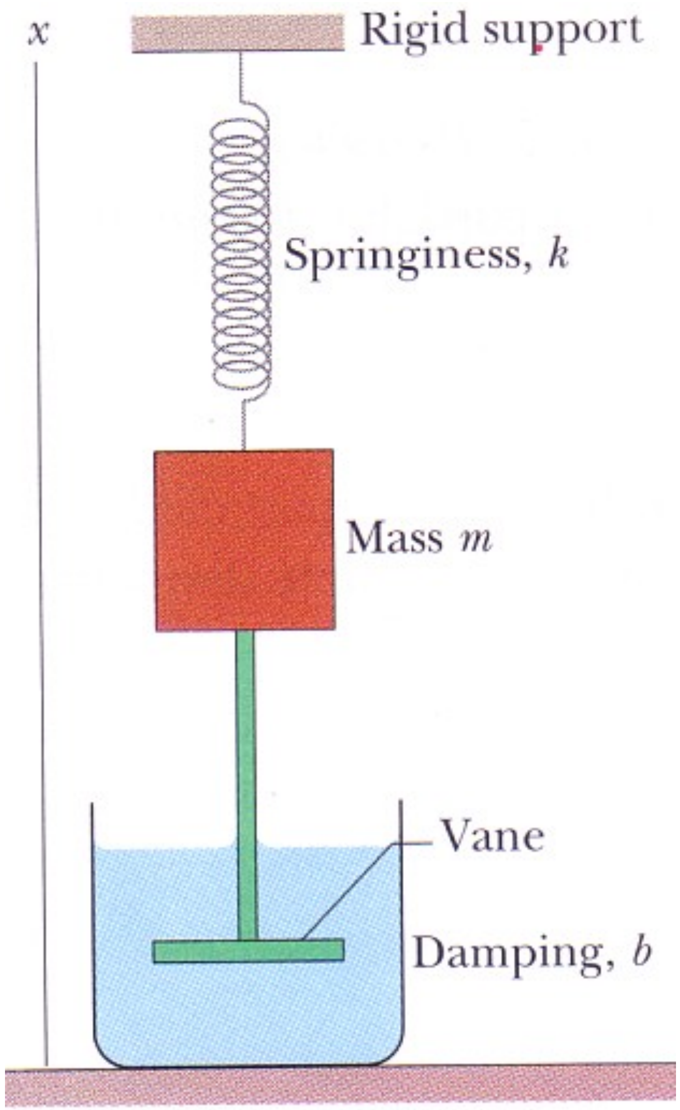
- The force due to the spring is still $-kx$. Thus,

$$-bv - kx = ma$$

Or

$$m\frac{d^2x}{dx^2} + b\frac{dx}{dt} + kx = 0$$

Damped Simple Harmonic Motion



- When the motion of an oscillator is reduced by an external force, the oscillator and its motion are said to be damped.
- Let us assume that the liquid in the figure (left) exerts a constant damping force that is proportional in magnitude to the velocity (like air resistance), and opposite in sign.
- The solution to

$$m\frac{d^2x}{dx^2}+b\frac{dx}{dt}+kx=0$$

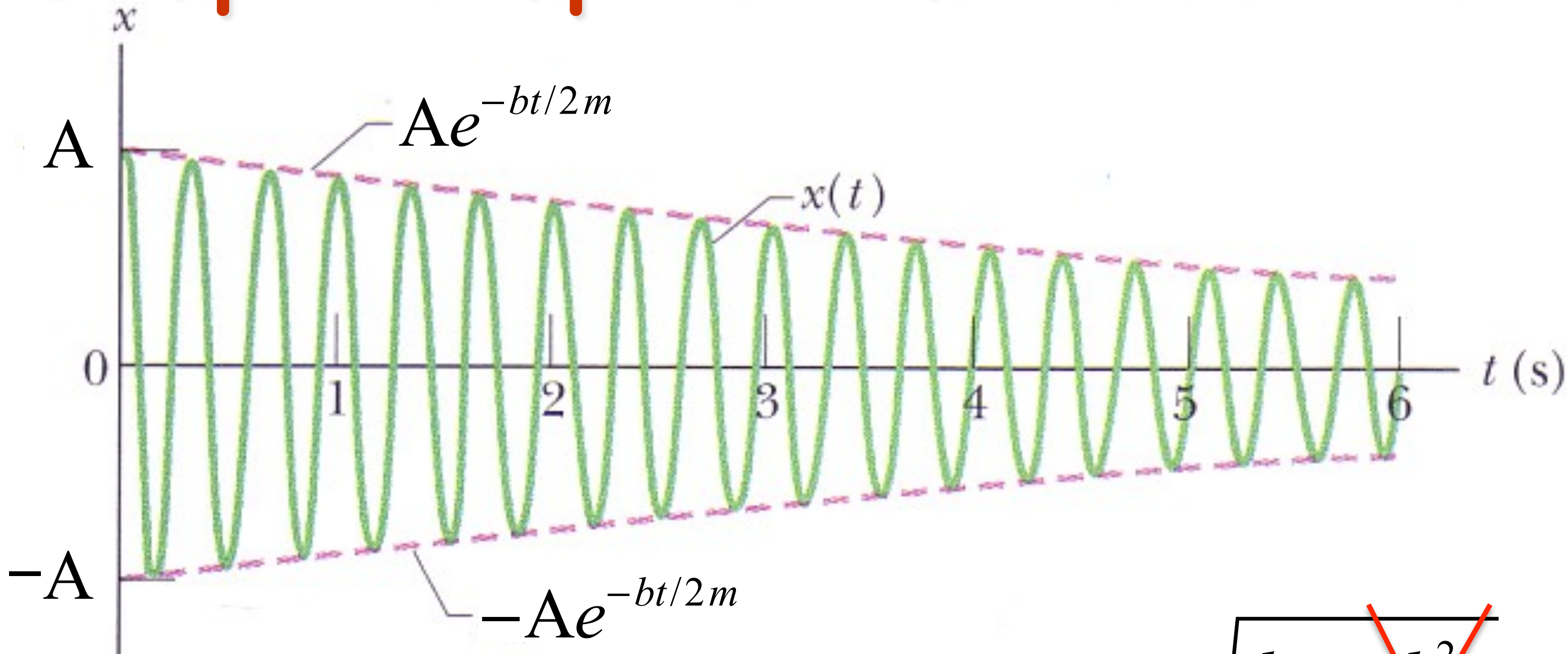
is

$$x(t)=\mathrm{A}e^{-bt/2m}\cos(\omega' t+\phi)$$

where

$$\omega'=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$$

Damped Simple Harmonic Motion



$$x(t) = Ae^{-bt/2m}\cos(\omega' t + \phi)$$

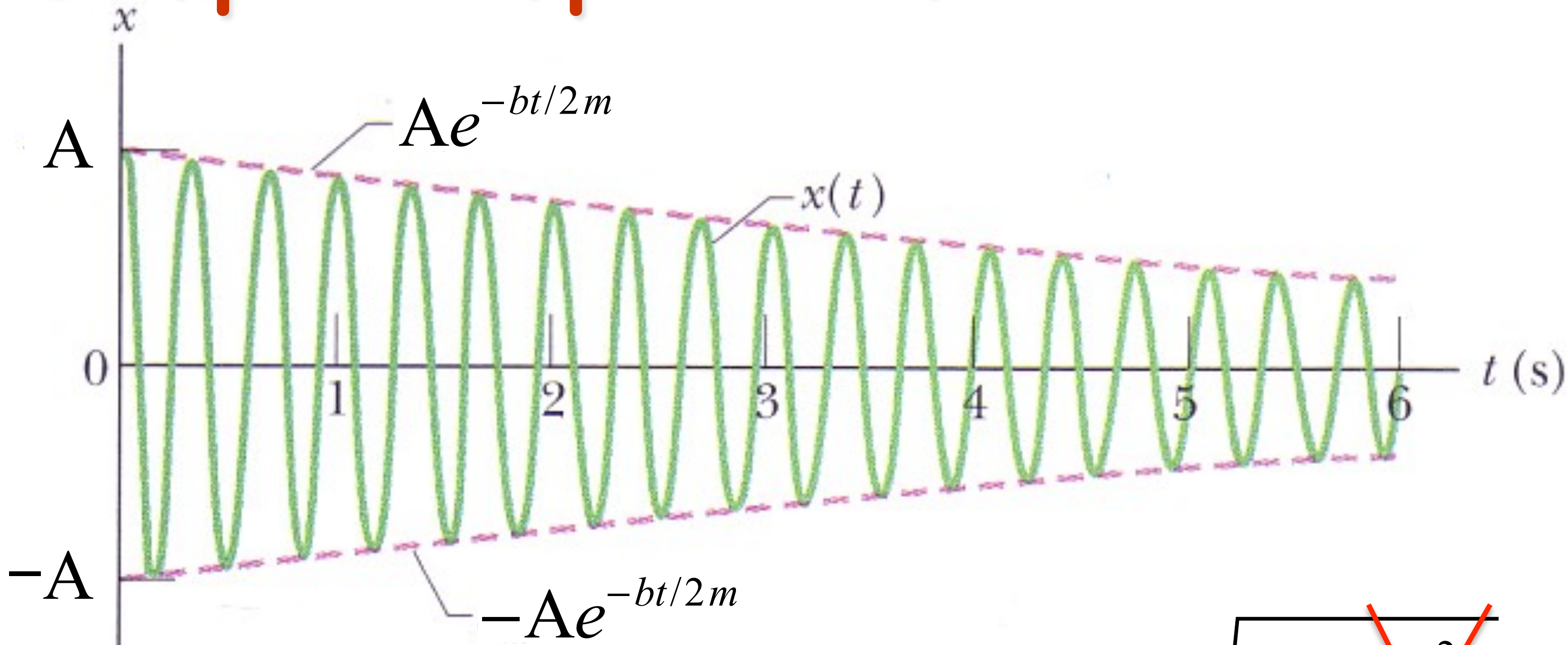
$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

- If $(b^2/4m^2) << (k/m)$, i.e., $b << (km)^{1/2}$, then $\omega' \approx \omega$

Then:

$$x(t) \approx Ae^{-\alpha t}\cos(\omega t + \phi) \qquad [\alpha = b/2m]$$

Damped Simple Harmonic Motion



$$x(t) = Ae^{-bt/2m}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

- If $(b^2/4m^2) << (k/m)$, *i.e.*, $b << (km)^{1/2}$, then $\omega' \approx \omega$
- The mechanical energy is then given by:

$$E(t) \approx \tfrac{1}{2}kA^2\left(e^{-\alpha t}\right)^2 = \tfrac{1}{2}kA^2e^{-2\alpha t} = E_m e^{-2\alpha t}$$